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Promotionsverfahren von **Herrn M.Sc. Erik Sebastian Chudzik**
Auslage der Dissertation und Gutachten sowie Termin der mündlichen Prüfung
Anlage: Einseitige Zusammenfassung der Dissertation

Sehr geehrte Damen und Herren,

in dem oben genannten Promotionsverfahren wird die Annahme der Dissertation

Cartesian Grid Active Flux Methods for Hyperbolic Conservation Laws

von den Berichterstattenden Prof. Dr. C. Helzel und Prof. Dr. D. Calhoun beantragt. Sie kann zusammen mit den Gutachten in der Zeit

vom 31.05.2024 bis 11.06.2024

eingesehen werden. Bitte wenden Sie sich zur Einsicht an das Promotionsbüro (promotionmnf@hhu.de).

Einsprüche gegen diese Dissertation können nur zwei Tage nach der vorgenannten Frist geltend gemacht werden. Erfolgt kein Einspruch, so gilt die Dissertation als angenommen (§ 7 Ziffer (5) PO).

Sofern die Dissertation angenommen wird, findet die mündliche Prüfung am

14.06.2024 um 14:30 Uhr

im **Raum 25.22.00.53** statt. Als Prüferinnen bzw. Prüfer sind vorgesehen:
Prof. Dr. M. Lukácová-Medvidová, Prof. Dr. F. Jarre und Priv.-Doz. Dr. A. Rätz.

Die Öffentlichkeit ist bei der Befragung nicht zugelassen.

Mit freundlichen Grüßen
im Auftrag

Amina Diekmann

Summary of the thesis "Cartesian Grid Active Flux Methods for Hyperbolic Conservation Laws" presented by Erik Chudzik

The Active Flux method is novel a finite volume method for hyperbolic conservation laws. We will consider the method on Cartesian grids. A distinct feature of the Active Flux method is the use of not only cell averages but also point values along grid cell interfaces. Using these point values and the cell averages, we reconstruct a continuous piecewise quadratic function. The resulting method is then third-order accurate, truly multi-dimensional, fully discrete and has a compact stencil in space and time. Furthermore, the method gives accurate approximations even on coarse grids.

This thesis is cumulative and consists of an introduction, which also contains new results, and two attached papers. It is organised as follows.

After introducing the method in one, two and three spatial dimensions, we perform a linear stability analysis of the derived one- and two-dimensional method for linear advection. However, in practise the equations of interest e.g., the Euler equations of gas dynamics, are non-linear. High order accurate methods for nonlinear problems require some form of limiting in order to approximate e.g., shock waves. First, we have to limit the reconstruction in order not to introduce artificial extrema when evolving the point values. Secondly, we have to limit the numerical fluxes to avoid under- or overshoots in the cell averages. We therefore review a bound preserving reconstruction limiter and a positivity preserving flux limiter. This is finally illustrated by numerical results. Attached are then the two papers.

We implemented an Active Flux method in ForestClaw, which is a software for patch-based adaptive mesh refinement on a forest of quadtrees. Adaptive mesh refinement allows us to resolve regions of solution structures, which are particularly interesting, more accurate. It turns out that our methods is well suited for the use of adaptive mesh refinement. This adaptively refined Active Flux method is described in detail in the first attached paper with the title "The Cartesian Grid Active Flux Method with Adaptive Mesh Refinement".

Though the Active Flux method is a finite volume method i.e., the cell averages of the conserved variables are the quantities of interest, a crucial point is how to evolve the point values in time with sufficiently high order of accuracy. The method of bicharacteristics provides a general framework for the construction of evolution operators for linear hyperbolic systems, which we used in the context of two-dimensional Active Flux methods for hyperbolic systems. A detailed description of these newly derived Active Flux methods can be found in the second attached paper with the title "Active Flux Methods for Hyperbolic Systems using the Method of Bicharacteristics".